1)(20 points) A piston filled with 0.5 moles of an ideal gas expands reversibly from 150.0 mL to 1750 mL at a constant temperature of 27.0 °C. Calculate q, w, ΔU and ΔH for the process. Show all work.

Isothermal process ΔT = 0

ΔU = \( C_v \Delta T = 0 \)
ΔH = \( C_p \Delta T = 0 \)

\[ w = -pdV = -nRT \ln \left( \frac{V_f}{V_i} \right) = -0.5 \text{ mol} \times 8.314 \text{ J/mol-K} \times 273 \text{ K} \times \ln \left( \frac{1750}{150} \right) = -3064 \text{ J} \]

ΔU = 0 = q + w
q = - w = 3064 J
2) (20 points) 1 mole of an ideal gas at an initial pressure of 5 atm and initial temperature of 273.15 K is allowed to expand adiabatically against a pressure of 2.25 atm until its volume doubles. Calculate the work, final temperature, $\Delta U$ and $\Delta H$ for this process.

\[ q = 0 \]

\[ V_i = 1 \text{mol} \times 0.082 \text{ L-atm/mol-K} \times 273/5 \text{ atm} = 4.48 \text{ L} \]

\[ V_f = 8.96 \text{ L} \]

\[ \Delta U = w = -pdV \]

\[ = -2.25 \text{ atm} \times (8.96 - 4.48 \text{ L}) = -10.08 \text{ L-atm} \times 101.325 \text{ J/L-atm} = -1.02 \text{ kJ} \]

\[ \Delta H = \Delta U + \Delta(PV) = -1.02 \text{ kJ} + (2.25 \text{ atm} \times 8.96 \text{ L} - 5 \text{ atm} \times 4.48 \text{ L}) \times 101.325 \text{ J/L-atm} \]

\[ = -1.25 \text{ kJ} \]

\[ T_f = PV/nR = 2.25 \text{ atm} \times 8.96 \text{ L}/(1 \text{ mol} \times 0.082 \text{ L-atm/mol-K}) = 245 \text{ K} \]
3) (20 points)
   a) What is the total differential of the function: \( z = 3x^3 - 4xy^2 - 115 \)?
   b) Show that the function is exact.
   c) Why does a state function have to be an exact function.

a)
   \[
   dz = \left( \frac{\partial z}{\partial x} \right)_y dx + \left( \frac{\partial z}{\partial y} \right)_x dy
   \]
   \[
   \left( \frac{\partial z}{\partial x} \right)_y = 9x^2 - 4y^2 \\
   \left( \frac{\partial z}{\partial y} \right)_x = -8xy
   \]
   \[
   dz = (9x^2 - 4y^2)dx + (-8xy)dy
   \]

b) \[
   \left( \frac{\partial^2 z}{\partial y \partial x} \right)_x = \left( \frac{\partial (9x^2 - 4y^2)}{\partial y} \right)_x = -8y \\
   \left( \frac{\partial^2 z}{\partial x \partial y} \right)_y = \left( \frac{\partial (-8xy)}{\partial x} \right)_y = -8y
   \]
   Since \( \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y} \) the function is exact.

c) State functions are route independent. They should give the same answer if the system goes in the x direction first and then the y direction or the y direction first and then the x direction as long as the system arrives in the same final state.
4)(20 points) Determine which of the two gas models, ideal and hard sphere (the van der Waal’s model with $a = 0$ & $b > 0$) has the smallest expansion coefficient, $\alpha$.

$$\alpha = \frac{1}{V} \frac{\partial V}{\partial T} p$$

**Ideal Gas**

$$V = nRT/p, \quad \frac{\partial V}{\partial T} p = nR/p$$

$$\alpha = \frac{1}{V} \frac{\partial V}{\partial T} p = \frac{1}{V} nR/p = nR/pV = 1/T$$

**Hard Sphere Gas**

$$p = (nRT/(V-b); \quad V = nRT/p + b, \quad \frac{\partial V}{\partial T} p = nR/p$$

$$\alpha = \frac{1}{V} \frac{\partial V}{\partial T} p = \frac{1}{V} nR/p = nR/pV = nR/p(nRT/p + b)$$

$$\alpha = \frac{1}{T+(bp/nR)}$$

The hard sphere gas will have the smaller expansion coefficient as $1/T > 1/(T+(bp/nR))$ since $n, R, T & b > 0$. 

5)(20 points) Show that the Joule-Thompson Coefficient $\mu$ can be written as:

$$\mu = -(1/C_p)\mu_T$$

$$\mu = (?T/?p)_H$$

Using Euler Relation:

$$ (?T/?p)_H (?p/?H)_T (?H/?T)_P = -1 $$

$$ (?T/?p)_H = - (?H/?p)_T (?T/?H)_P $$

$\mu_T$ is defined as $(?H/?p)_T$

$$ (?T/?p)_H = - \mu_T (?T/?H)_P $$

$C_p$ is defined as $(?H/?T)_P$

$$ \mu = - \mu_T (1/C_p) $$