Maxwell Relations

First Law: \( dU = dq + dw \)
Definition of Work: \( dw = -pdv \)
Definition of Heat: \( dq = TdS \)

\[
dU = TdS - pdV \\
dH = TdS + VdP \quad \text{since} \quad dH = dU + d(pV) = TdS - pdV + pdV + Vdp \\
dA = -pdV - SdT \quad \text{since} \quad dA = dU - d(TS) = TdS - pdV - TdS - SdT \\
dG = VdP - SdT \quad \text{since} \quad dG = dH - d(TS) = TdS + VdP - TdS - SdT
\]

Natural Variables

\[
U = U(S, V) \\
H = H(S, P) \\
A = A(V, T) \\
G = G(P, T)
\]

From the definition of the differential \( df(x, y) = (\partial f/\partial x)_y dx + (\partial f/\partial y)_x dy \)

\[
T = (\partial U/\partial S)_V = (\partial H/\partial S)_p \\
-P = (\partial U/\partial V)_S = (\partial A/\partial V)_T \\
V = (\partial H/\partial P)_S = (\partial G/\partial P)_T \\
-S = (\partial A/\partial T)_V = (\partial G/\partial T)_P
\]

Based on conditions for exactness of state functions \( dz = dfx + gdy \) then \( (\partial f/\partial y)_x = (\partial g/\partial x)_y \)

\[
dU = TdS - pdV \Rightarrow (\partial T/\partial V)_S = - (\partial p/\partial S)_V \\
dH = TdS + VdP \Rightarrow (\partial T/\partial p)_S = (\partial V/\partial S)_P \\
dA = -pdV - SdT \Rightarrow (\partial p/\partial T)_V = (\partial S/\partial V)_T \\
dG = VdP - SdT \Rightarrow (\partial V/\partial T)_P = - (\partial S/\partial P)_T
\]

Other useful tools:
Chain Rule

\[
(\partial f/\partial t)_S = (\partial f/\partial g)(\partial g/\partial t)
\]

Euler’s Rule for \( z = z(x, y) \) aka Cyclic Relation for Partial Derivatives

\[
(\partial y/\partial x)_z(\partial x/\partial z)_y(\partial z/\partial y)_x = -1
\]

Product Rule

\[
(\partial (fg)/\partial x) = f(\partial g/\partial x) + g(\partial f/\partial x)
\]